Exam Advanced Logic

April 3rd, 2019

Instructions:

- Put your student number on the first page and subsequent pages (not your name, in order to allow for anonymous grading).
- Do not use pencil (or an erasable pen) or a red pen to make your exam.
- Motivate all your answers.
- Please leave open 10 lines on the first page to give us room for writing down the points.
- Your exam grade is computed as min(10, (the sum of all your points + 10) divided by 10). For example, someone who would only fill in a student number would get a 1, while someone who would make questions 1-9 perfectly but would skip the bonus question would get min(10, (90+10)/10) = 10.

Good luck!

1. **Induction (10 pt)**

   Let $\mathscr{L}_{\leftrightarrow}$ be an alternative language of propositional logic based on the operator $\leftrightarrow$ only (so without $\neg$, $\wedge$, $\vee$, $\rightarrow$).

   (a) Give an inductive definition of the well-formed formulas of $\mathscr{L}_{\leftrightarrow}$.

   (b) Give an inductive definition of $v(A)$, the truth value of formula A in the language $\mathscr{L}_{\leftrightarrow}$ under valuation v.

   (c) Let valuation v_1 be given such that $v_1(p) = 1$ for all propositional parameters p. Prove by induction that $v_1(A) = 1$ for all formulas A in $\mathscr{L}_{\leftrightarrow}$.

   (d) Is $\{\leftrightarrow\}$ functionally complete, i.e. is it the case that every formula of propositional logic is equivalent to a formula in $\mathscr{L}_{\leftrightarrow}$? Explain your answer.

2. **Three-valued logics (10 pt)** Using a truth table, determine whether the following inference holds in $\mathbf{Ł}_3$:

   $$\models_{Ł_3} (p \supset (\neg q \vee q)) \vee ((\neg q \vee q) \supset p)$$

   Write out the full truth table and do not forget to draw a conclusion.

3. **Tableaux for FDE and related many-valued logics (10 pt)** By constructing a suitable tableau, determine whether the following inference is valid in $\mathbf{K_3}$. If the inference is invalid, provide a counter-model.

   $$p \wedge ((\neg p \vee q) \wedge (\neg q \vee r)) \vdash_{K_3} p \wedge q$$

   NB: Do not forget to draw a conclusion from the tableau.

4. **Fuzzy logic (10 pt)** Determine whether the following holds in the fuzzy logic with set of designated values $D_{0.8} = \{x : x \geq 0.8\}$. If so, explain why. If not, provide a counter-model and show why it is one.

   $$(r \rightarrow q) \rightarrow (p \rightarrow r) \models_{0.8} p \rightarrow r$$

5. **Basic modal tableau (10 pt)** By constructing a suitable tableau, determine whether the following is valid in K. If the inference is invalid, provide a counter-model.

   $$\Box\Box q \vdash_K \Box(p \supset \Box q) \wedge (\Diamond p \supset \Diamond\Box q)$$

   NB: Do not forget to draw a conclusion from the tableau.

6. **Normal modal tableau (10 pt)** By constructing a suitable tableau, determine whether the following tense-logical inference is valid in K^t_τ (transitive). If the inference is invalid, provide a counter-model.

$$\langle F \rangle q \vdash_{K^t_\tau} \langle F \rangle (q \wedge [F]\neg q)$$

NB: Do not forget to draw a conclusion from the tableau.

7. **Soundness and completeness (10pt)** As a reminder, the rule φ for tense logic is:

$$\begin{array}{ccc} & irj & \\ & irk & \\ \swarrow & \downarrow & \searrow \\ jrk & j = k & krj \end{array}$$

And the auxiliary rules for $=$ (that are included in the tense logic tableau system K^t_φ) are as follows, where α is a formula of the temporal language:

$$\begin{array}{c} \alpha(i) \\ i = j \\ \downarrow \\ \alpha(j) \end{array} \qquad \begin{array}{c} \alpha(i) \\ j = i \\ \downarrow \\ \alpha(j) \end{array}$$

Let b be a complete open branch of a K^t_φ-tableau, and let $I = \langle W, R, v \rangle$ be an interpretation that is *induced* by b. Show that the accessibility relation R of I is *forward convergent* , that is, for all $x, z, y \in W$, if xRy and xRz, then (zRy or $y = z$ or yRz).

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8. **First-order modal tableau, variable domain (10 pt)** By constructing a suitable tableau, determine whether the following is valid in VK. If the inference is invalid, provide a counter-model.

$$\forall x \Box \forall y Pxy \vdash_{VK} \forall x \forall y \Box Pxy$$

NB: Do not forget to draw a conclusion from the tableau.

9. **Default logic (10 pt)**

Consider the following set of default rules, where p, q, r, s are propositional atoms:

$$D = \left\{ \delta_1 = \frac{p : q \wedge r}{s}, \qquad \delta_2 = \frac{p : q \wedge \neg r}{\neg r}, \qquad \delta_3 = \frac{s : \neg q}{\neg q} \right\},$$

and initial set of facts:

$$W = \{p\}.$$

This exercise is about the default theory $T = (W, D)$.

(a) Of each of the following sequences, state whether it is a *process*; and if so, whether or not the process is *closed*, and whether or not it is *successful*. Briefly explain your answers.

i. (δ_1)

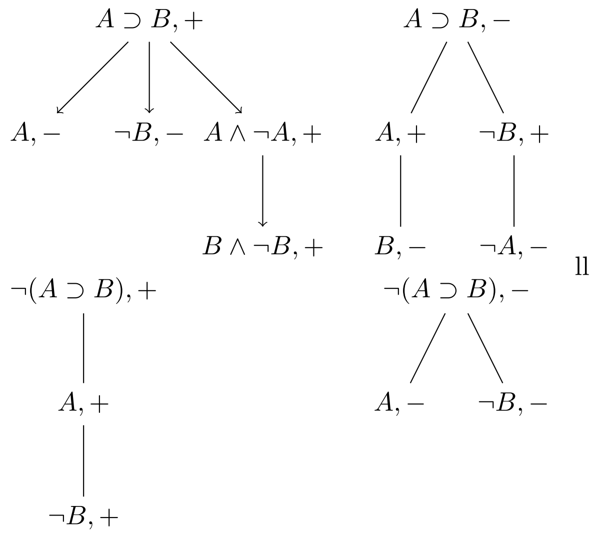
ii. (δ_1, δ_2)

(b) Draw the process tree of the default theory (W, D).

(c) What are the extensions of (W, D)?

(d) Is $q \wedge s$ a credulous consequence of (W, D)? Explain.

10. **Bonus (10 pt)** As a reminder, the RM_3 tableau rules for $\supset$ are as follows



Consider the language of FDE, containing the connectives $\neg, \vee, \wedge$. Does the following statement hold for all wffs A, B, C in that language?

"If $A \vdash_{K_3} \neg B \vee C$ and $A \vdash_{RM_3} B \supset C$, then $A \vdash_{FDE} \neg B \vee C$"

If yes, please explain exactly why the statement holds for all wffs A, B, C in the language of FDE. If no, please provide a triple of wffs A, B, C and show in detail why that triple forms a counterexample to the statement.